CLASS 12

FOR CBSE EXAMS
(2026-27)

SOLUTIONS
OF MATHMISSION

01 DETAILED STEP-BY-STEP SOLUTIONS
- Suitable for Step-wise Marking in the CBSE 2027 Board Exams
- Easy to understand approach to Problem-solving

02 SHORT - TRICKS & TECHNIQUES
- Useful for Multiple Choice Questions / Assertion-Reason Questions
- To save time in the exams

03 KEY FORMULAE / IDENTITIES / THEOREMS & TIPS RECALLED
- To Build conceptual clarity & boost confidence

04 DIAGRAMS & GRAPHS TO AID IN LEARNING
- Helpful in visualization & enhances memory recall in exams

05 ALTERNATIVE SOLUTIONS WHEREVER POSSIBLE
- To develop critical thinking & to foster creativity and flexibility

O.P. GUPTA
INDIRA AWARD WINNER

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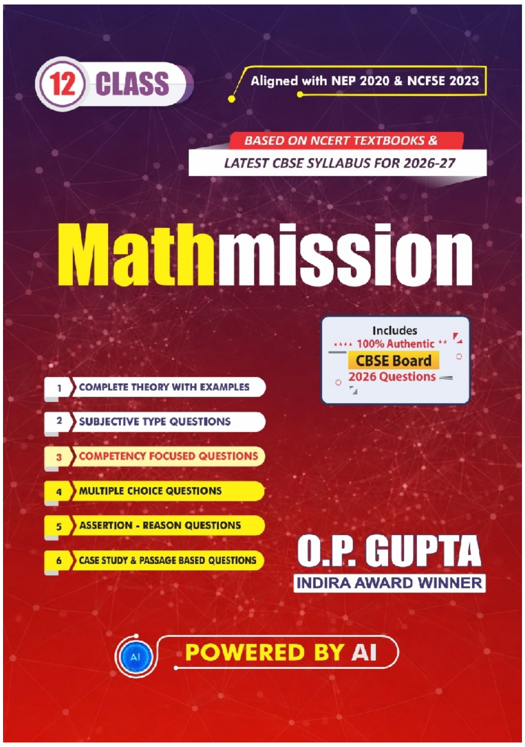

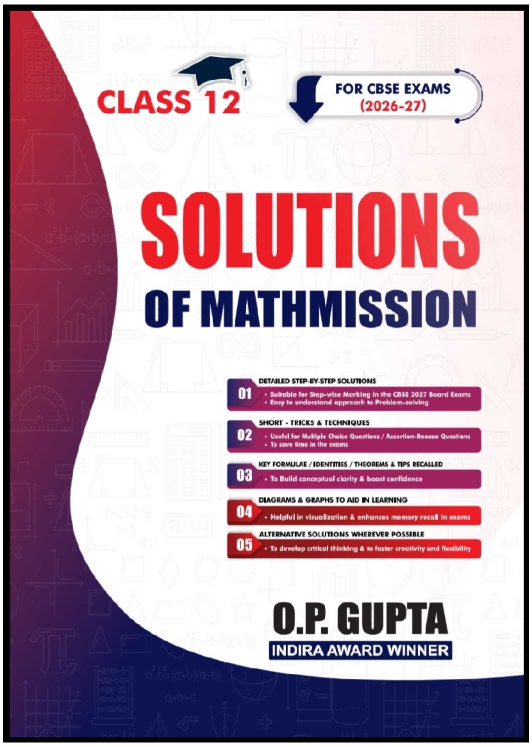

Following are the two Books for CBSE XII (2026-27) by O.P. Gupta, released in March 2026.

🕮 **MATHMISSION FOR XII (2026-27)**
For CBSE Board Exams ▪ Maths (041)
By O.P. Gupta (Indira Award Winner)

- ✪ **Detailed Theory with Examples**
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- ✪ **Selected H.O.T.S. Questions (from recent CBSE 2026 & 2025 Exams)**
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🕮 **SOLUTIONS OF MATHMISSION FOR XII (2026-27)**
For CBSE Board Exams ▪ Maths (041)
By O.P. Gupta (Indira Award Winner)

- ✪ **Step-by-Step Detailed Solutions of all the Exercises of MATHMISSION FOR XII**

ⓘ Books are available on Flipkart / Amazon / Theopgupta.com
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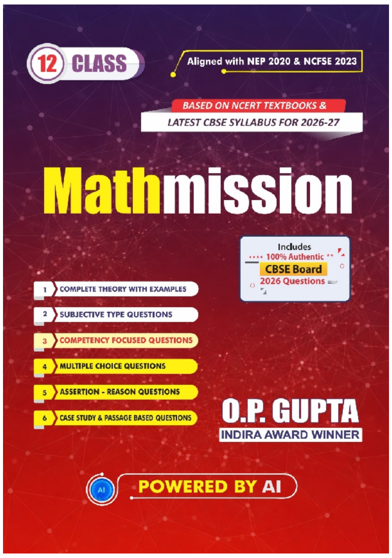

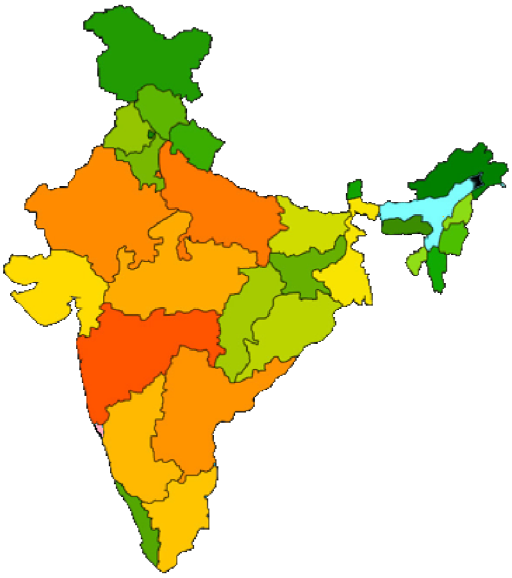

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- Oman
- Doha (Qatar)
- Saudi Arabia
- Dubai
- Singapore

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DETAILED SOLUTIONS

CHAPTER 01

EXERCISE 1.1

Q01. Possible orders : $1\times12,\ 2\times6,\ 3\times4,\ 4\times3,\ 6\times2,\ 12\times1$.

Q02. **(a)** Let A be the matrix of order 2×3

$$\therefore A=\left[a_{ij}\right]_{2\times3}=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\end{bmatrix}$$

Since there are 2 choices $(0 \text{ or } 1)$ to fill each places a_{ij} and repetition is allowed as well.

$\therefore$ Total no. of matrices $=2^6=64$.

(b) Let A be the matrix of order 3×3

$$\therefore A=\left[a_{ij}\right]_{3\times3}=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}$$

Since we have 2 entries $(0 \text{ and } 1)$ to fill 9 places of a_{ij} and repetitions is allowed as well.

$\therefore$ Total no. of all possible matrices $=2^9=512$.

(c) No. of all possible matrices of order 2×2 with each entry 1, 2 or 3 is 3^4 or 81.

Q03. **(a)** Let A be the matrix of order 4×3, so $A=\left[a_{ij}\right]_{4\times3}=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\\ a_{41} & a_{42} & a_{43}\end{bmatrix}$.

Given that $a_{ij}=\dfrac{i-j}{i+j}$

$$\therefore a_{11}=\frac{1-1}{1+1}=0,\quad a_{12}=\frac{1-2}{1+2}=\frac{-1}{3},\quad a_{13}=\frac{1-3}{1+3}=\frac{-1}{2},$$

$$a_{21}=\frac{2-1}{2+1}=\frac{1}{3},\quad a_{22}=\frac{2-2}{2+3}=0,\quad a_{23}=\frac{2-3}{2+3}=\frac{-1}{5},$$

$$a_{31}=\frac{3-1}{3+1}=\frac{1}{2},\quad a_{32}=\frac{3-2}{3+2}=\frac{1}{5},\quad a_{33}=\frac{3-3}{3+3}=0,$$

$$a_{41}=\frac{4-1}{4+1}=\frac{3}{5},\quad a_{42}=\frac{4-2}{4+2}=\frac{1}{3},\quad a_{43}=\frac{4-3}{4+3}=\frac{1}{7}$$

$$\therefore A=\begin{bmatrix}0 & -1/3 & -1/2\\ 1/3 & 0 & -1/5\\ 1/2 & 1/5 & 0\\ 3/5 & 1/3 & 1/7\end{bmatrix}.$$

(b) Let B be the matrix of order 3×2.

Given that $[b_{ij}]=\dfrac{|i-2j|}{3}$.

Assume that, $B=\left[b_{ij}\right]_{3\times2}=\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix}$

$\therefore b_{11}=\frac{|1-2\times1|}{3}=\frac{1}{3},\ b_{12}=\frac{|1-2\times2|}{3}=1,\ b_{21}=\frac{|2-2\times1|}{3}=0,\ b_{22}=\frac{|2-2\times2|}{3}=\frac{2}{3},$

$b_{31}=\frac{|3-2\times1|}{3}=\frac{1}{3},\ b_{32}=\frac{|3-2\times2|}{3}=\frac{1}{3}$

$\therefore B=\left[b_{ij}\right]_{3\times2}=\begin{bmatrix} 1/3 & 1 \\ 0 & 2/3 \\ 1/3 & 1/3 \end{bmatrix}.$

(c) Let A be the matrix of order 2×3.

$\therefore A=\left[a_{ij}\right]_{2\times3}=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$

$\therefore a_{11}=1-1=0,\ a_{12}=-1+3(2)=5,\ a_{13}=-1+3(3)=8,$

$a_{21}=2-2(1)=0,\ a_{22}=2-2=0,\ a_{23}=-2+3(3)=7$

$\therefore A=\left[a_{ij}\right]_{2\times3}=\begin{bmatrix} 0 & 5 & 8 \\ 0 & 0 & 7 \end{bmatrix}.$

Q04. **(a)** $A=\lambda\left[a_{ij}\right]_{3\times3}$ and, $a_{ij}=\frac{2(9i-j)}{3}$

$\therefore a_{23}=\lambda\left[a_{23}\right]=\lambda\frac{2[9\times2-3]}{3}=10\lambda$

(b) Elements of $A=\left[a_{ij}\right]_{2\times2}$ are given by $a_{ij}=\frac{i}{j}$

$\therefore a_{12}=\frac{1}{2}.$

(c) We have $a_{ij}=\frac{|i-j|}{2}$

$\therefore a_{23}=\frac{|2-3|}{2}=\frac{|-1|}{2}=\frac{1}{2}.$

(d) Since $a_{ij}=e^{2ix}\sin jx$ so, $a_{12}=e^{2x}\sin 2x$.

Q05. $A=\begin{bmatrix} 3 & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$\because$ Matrix A is a scalar matrix.

$\therefore$ $x=3$ (using definition).

Q06. According to question, $A=\begin{bmatrix} \cos\omega & -\sin\omega \\ \sin\omega & \cos\omega \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now by equality of matrices, we get : $\cos\omega=1,\ \sin\omega=0$

$\Rightarrow \cos\omega=\cos 0,\ \sin\omega=\sin 0$

$\therefore \omega=2n\pi,\ n\in Z$ and $\omega=n\pi,\ n\in Z$.

Therefore, $\omega = 2n\pi,\ n \in Z$.

Q07. We have $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$$\therefore (3A - B) = 3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

Therefore, $(3A - B) = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$.

Q08. $A = \text{diag}\begin{bmatrix} 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ and, $B = \text{diag}\begin{bmatrix} 2 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

$$\therefore 3A + 4B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 6 \end{bmatrix} + \begin{bmatrix} 8 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

$$\Rightarrow 3A + 4B = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Therefore, $3A + 4B = \text{diag}\begin{bmatrix} 11 & 9 & 2 \end{bmatrix}$.

Q09. We have $\cos\omega\begin{bmatrix} \cos\omega & \sin\omega \\ -\sin\omega & \cos\omega \end{bmatrix} + \sin\omega\begin{bmatrix} \sin\omega & -\cos\omega \\ \cos\omega & \sin\omega \end{bmatrix}$

$$\Rightarrow \quad = \begin{bmatrix} \cos^2\omega & \cos\omega\sin\omega \\ -\cos\omega\sin\omega & \cos^2\omega \end{bmatrix} + \begin{bmatrix} \sin^2\omega & -\sin\omega\cos\omega \\ \sin\omega\cos\omega & \sin^2\omega \end{bmatrix}$$

$$\Rightarrow \quad = \begin{bmatrix} \cos^2\omega + \sin^2\omega & \cos\omega\sin\omega - \cos\omega\sin\omega \\ -\cos\omega\sin\omega + \cos\omega\sin\omega & \cos^2\omega + \sin^2\omega \end{bmatrix}$$

$$\Rightarrow \quad = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I.$$

Q10. Since $2A + 3X = 5B$

$$\Rightarrow 2\begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix} + 3X = 5\begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 16 & 0 \\ 8 & -4 \\ 6 & 12 \end{bmatrix} + 3X = \begin{bmatrix} 10 & -10 \\ 20 & 10 \\ -25 & 5 \end{bmatrix}$$

$$\Rightarrow 3X = \begin{bmatrix} -6 & -10 \\ 12 & 14 \\ -31 & -7 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{3}\begin{bmatrix} -6 & -10 \\ 12 & 14 \\ -31 & -7 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} -2 & -\frac{10}{3} \\ 4 & \frac{14}{3} \\ -\frac{31}{3} & -\frac{7}{3} \end{bmatrix}.$$

Q11. $\begin{bmatrix} -2 & 4 & -2 \\ 3 & 7 & 3 \end{bmatrix} + A = \begin{bmatrix} -1 & 2 & 6 \\ 4 & 5 & 0 \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} -1 & 2 & 6 \\ 4 & 5 & 0 \end{bmatrix} - \begin{bmatrix} -2 & 4 & -2 \\ 3 & 7 & 3 \end{bmatrix}$

$\therefore A = \begin{bmatrix} 1 & -2 & 8 \\ 1 & -2 & -3 \end{bmatrix}.$

Q12. **(a)** $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+a \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

By equality of matrices, $x-y=-1$ ...(i), $2x+z=5$ ...(iii)

$2x-y=0$ ...(iii), $3z+a=13$ ...(iv)

By (i) and (iii), $x-2x=-1 \Rightarrow x=1$ $\therefore y=2$

Replacing value of x in (iii), $2(1)+z=5 \Rightarrow z=3$; also, $3(3)+a=13 \Rightarrow a=4$

$\therefore x=1,\ y=2,\ z=3,\ a=4$.

(b) $\begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix} = 2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix} = \begin{bmatrix} 2x+3 & 14 \\ 15 & 2y-4 \end{bmatrix}$

By equality of matrices, $2x+3=7 \Rightarrow x=2$ and, $2y-4=14 \Rightarrow y=9$

$\therefore x=2,\ y=9$.

(c) $\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - 3\begin{bmatrix} x \\ 2y \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - \begin{bmatrix} 3x \\ 6y \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x^2-3x \\ y^2-6y \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \end{bmatrix}$

By equality of matrices, we get :

$x^2-3x=-2,\ y^2-6y=9$ $\Rightarrow x^2-3x+2=0$...(i), $y^2-6y-9=0$...(ii)

By (i), $x^2-3x+2=0$ $\Rightarrow x^2-2x-x+2=0$ $\Rightarrow (x-1)(x-2)=0$

$\therefore x=1,\ 2$.

By (ii), $y^2-6y-9=0$

$\Rightarrow y = \frac{6\pm\sqrt{36+36}}{2} = \frac{6\pm\sqrt{72}}{2} = \frac{6\pm6\sqrt{2}}{2}$

$\therefore y = 3\pm3\sqrt{2}$.

(d) $\begin{bmatrix} x+3 & z+4 & 2y-7 \\ 4x+6 & a-1 & 0 \\ b-3 & 3b & z+2c \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ 2x & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$

By equality of matrices, $x+3=0 \Rightarrow x=-3,\ z+4=6 \Rightarrow z=2,\ 2y-7=3y-2 \Rightarrow y=-5,$
$a-1=-3 \Rightarrow a=-2,\ 3b=-21 \Rightarrow b=-7,\ 2c+2=0 \Rightarrow c=-1$
$\therefore a=-2,\ b=-7,\ c=-1,\ x=-3,\ y=-5,\ z=2$.

Q13. **(a)** Given $2\begin{pmatrix} 3 & 4 \\ 5 & x \end{pmatrix} + \begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 10 & 5 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 6 & 8 \\ 10 & 2x \end{pmatrix} + \begin{pmatrix} 1 & y \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 10 & 5 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 7 & 8+y \\ 10 & 2x+1 \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 10 & 5 \end{pmatrix}$

By equality of matrices : $8+y=0,\ 2x+1=5$
$\Rightarrow y=-8,\ x=2$.
Therefore, $(x-y)=2-(-8)=10$.

(b) Given that $A = \begin{pmatrix} 0 & 3 \\ 2 & -5 \end{pmatrix}$ and $kA = \begin{pmatrix} 0 & 4a \\ -8 & 5b \end{pmatrix}$

So, $k\begin{pmatrix} 0 & 3 \\ 2 & -5 \end{pmatrix} = \begin{pmatrix} 0 & 4a \\ -8 & 5b \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 0 & 3k \\ 2k & -5k \end{pmatrix} = \begin{pmatrix} 0 & 4a \\ -8 & 5b \end{pmatrix}$

By equality of matrices, $3k=4a,\ 2k=-8,\ -5k=5b$
Solving these equations simultaneously we get : $k=-4,\ a=-3$.

(c) $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2x+3 & 6 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

By def. of equality of matrices, we get $2x+3=7,\ 2y-4=14$
On adding these equations, we get : $(2x+3)+(2y-4)=7+14$
$\Rightarrow 2(x+y)=22$
$\therefore x+y=11$.

(d) Given $\begin{pmatrix} a+4 & 3b \\ 8 & -6 \end{pmatrix} = \begin{pmatrix} 2a+2 & b+2 \\ 8 & a-8b \end{pmatrix}$

By equality of matrices, $a+4=2a+2,\ 3b=b+2,\ -6=a-8b$
Solving these equations, we get : $a=2,\ b=1$
$\therefore a-2b=2-2(1)=0$.

EXERCISE 1.2

Q01. **(a)** Given $2A-B = \begin{bmatrix} 4 & -6 \\ -4 & 2 \end{bmatrix} \ldots(i),\ A+2B = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix} \ldots(ii)$

By 2(i) + (ii), $2(2A-B)+(A+2B)=\begin{bmatrix}8 & -12\\ -8 & 4\end{bmatrix}+\begin{bmatrix}-1 & 0\\ 1 & 1\end{bmatrix}$

$\Rightarrow 5A=\begin{bmatrix}7 & -12\\ -7 & 5\end{bmatrix}$

$\Rightarrow A=\begin{bmatrix}7/5 & -12/5\\ -7/5 & 1\end{bmatrix}$.

Also by (i) – 2 (ii), $(2A-B)-2(A+2B)=\begin{bmatrix}4 & -6\\ -4 & 2\end{bmatrix}-\begin{bmatrix}-2 & 0\\ 2 & 2\end{bmatrix}$

$\Rightarrow -5B=\begin{bmatrix}6 & -6\\ -6 & 0\end{bmatrix}$

$\Rightarrow B=\begin{bmatrix}-\frac{6}{5} & \frac{6}{5}\\ \frac{6}{5} & 0\end{bmatrix}$

(b) Given $A-B=\begin{bmatrix}1 & 2 & -1\\ 0 & -2 & 1\\ 3 & 2 & -3\end{bmatrix}$...(i) and $A+3B=\begin{bmatrix}0 & 1 & 2\\ -4 & 2 & -2\\ -1 & 0 & -5\end{bmatrix}$...(ii)

By $3\times$(i)+(ii), we get : $(3A-3B)+(A+3B)=3\begin{bmatrix}1 & 2 & -1\\ 0 & -2 & 1\\ 3 & 2 & -3\end{bmatrix}+\begin{bmatrix}0 & 1 & 2\\ -4 & 2 & -2\\ -1 & 0 & -5\end{bmatrix}$

$\Rightarrow 4A=\begin{bmatrix}3 & 6 & -3\\ 0 & -6 & 3\\ 9 & 6 & -9\end{bmatrix}+\begin{bmatrix}0 & 1 & 2\\ -4 & 2 & -2\\ -1 & 0 & -5\end{bmatrix}$

$\Rightarrow A=\frac{1}{4}\begin{bmatrix}3 & 7 & -1\\ -4 & -4 & 1\\ 8 & 6 & -14\end{bmatrix}$

$\therefore A=\begin{bmatrix}\frac{3}{4} & \frac{7}{4} & -\frac{1}{4}\\ -1 & -1 & \frac{1}{4}\\ 2 & \frac{3}{2} & -\frac{7}{2}\end{bmatrix}$.

Q02. **(a)** $\begin{bmatrix}x+y & 3\\ 7 & xy\end{bmatrix}=\begin{bmatrix}1 & 3\\ 7 & -12\end{bmatrix}$

By equality of matrices, $x+y=1$ …(i) and $xy=-12$ …(ii)

By (i) and (ii), $x(1-x)=-12$ $\Rightarrow x-x^2=-12$ $\Rightarrow x^2-x-12=0$

$\Rightarrow x^2-4x+3x-12=0$ $\Rightarrow (x+3)(x-4)=0$

$\therefore x=-3,\ x=4$

If $x=-3$, then $y=1-x=1+3=4$.

And, if $x=4$ then, $y=1-x=1-4=-3$.

(b) $\begin{bmatrix} 2x+y & 3y \\ 0 & y^2-5y \end{bmatrix}=\begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$

By equality of matrices, $2x+y=x+3$

$\Rightarrow x+y=3$ …(i),

$3y=y^2+2 \quad \Rightarrow y^2-3y+2=0$ …(ii)

By (ii), $y^2-3y+2=0$

$\Rightarrow y^2-2y-y+2=0 \quad \Rightarrow (y-1)(y-2)=0 \quad \Rightarrow y=1,\ 2$

If $y=1$, then $x+1=3 \quad \Rightarrow x=2$ …(A)

If $y=2$, then $x+2=3 \quad \Rightarrow x=1$

Note that y = 1 [from (A)] doesn't satisfy the matrix equation so, $x=1,\ y=2$.

EXERCISE 1.3

Q01. No. of rows in matrix $X=a+b$, No. of columns in Matrix $X=a+2$

No. of rows in matrix $Y=b+1$, No. of columns in matrix $Y=a+3$

Given that XY & YX both exist.

If XY exists, $a+2=b+1 \Rightarrow a-b=-1$ …(i)

If YX exists, $a+b=a+3 \Rightarrow b=3$

By (i), $a=-1+3=2$

$\therefore a=2$ and $b=3$.

Q02. Note that $(2 \quad 1 \quad 3)_{1\times3}\begin{pmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}_{3\times3}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}_{3\times1}=A$ so, order of matrix A is 1×1.

Q03. Given $A=\begin{bmatrix} 1 & 3 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix}$

$\therefore AB=\begin{bmatrix} 1 \ 3 \ 2 \end{bmatrix}_{1\times3}\begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix}_{3\times1}=\begin{bmatrix} 1\times6+3\times2+2\times3 \end{bmatrix}=\begin{bmatrix} 18 \end{bmatrix}_{1\times1}$.

Q04. Let $A=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$\therefore AB=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}=O$.

Q05. $A=\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$

$\therefore A^2=A.A=\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}=\begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix}=\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}=-I \qquad [\because i^2=-1$

Q06. $A=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

As $A^4=A^2.A^2$ …(i)

Now $A^2 = A.A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow A^4 = A^2.A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I.I$

$\therefore A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I.$

Q07. We have $A = \begin{bmatrix} 0 & 0 \\ -3 & 0 \end{bmatrix}$

As $A^{20} = \left(A^2\right)^{10}$

Now $A^2 = A.A = \begin{bmatrix} 0 & 0 \\ -3 & 0 \end{bmatrix}.\begin{bmatrix} 0 & 0 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

$\therefore A^{20} = \left(A^2\right)^{10} = (O)^{10} = O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$

Q08. **(a)** $[x \ \ 1]_{1\times2}\begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix}_{2\times2}\begin{bmatrix} x \\ 5 \end{bmatrix} = O$

$\Rightarrow [(x-2) \ \ -3]\begin{bmatrix} x \\ 5 \end{bmatrix} = O$

$\Rightarrow \left[x^2 - 2x - 15\right] = O$

$\Rightarrow \left[x^2 - 2x - 15\right] = [0]$

$\Rightarrow x^2 - 2x - 15 = 0$ (By equality of matrices

$\Rightarrow x^2 - 5x + 3x - 15 = 0$

$\Rightarrow (x+3)(x-5) = 0$

$\therefore x = -3, \ x = 5$

(b) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}[1 \ \ 5] = \begin{bmatrix} 3 & 7x+y \\ 2y & 10 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 3 & 15 \\ 2 & 10 \end{bmatrix} = \begin{bmatrix} 3 & 7x+y \\ 2y & 10 \end{bmatrix}$

By equality of matrices, we get : $2y = 2 \Rightarrow y = 1, \ 7x + y = 15 \Rightarrow 7x = 14 \Rightarrow x = 2$

$\therefore x = 2$ and $y = 1$.

(c) $[1 \ \ 2 \ \ 1]\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$

$\Rightarrow [1 \ \ 2 \ \ 1]\begin{bmatrix} 0+4+0 \\ 0+0+x \\ 0+0+2x \end{bmatrix} = O$

$\Rightarrow [4 + 2x + 2x] = O$

$\Rightarrow [4 + 4x] = [0]$

By equality of matrices, we get : $4 + 4x = 0$

$\therefore x = -1$.

(d) $\begin{bmatrix} x & 2 & -1 \end{bmatrix}\begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 0 \\ 2 & 2 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = O$

$\Rightarrow \begin{bmatrix} 2x+2-2 & x+6-2 & -x-4 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = [0]$

$\Rightarrow \begin{bmatrix} 2x & x+4 & -(x+4) \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = [0]$

$\Rightarrow [2x + x + 4 - (x+4)] = [0]$

$\Rightarrow 2x = 0 \quad \therefore x = 0$.

Q09. $A = \begin{bmatrix} \omega & \kappa \\ \eta & -\omega \end{bmatrix}$ and, $A^2 = I$

$\therefore A^2 = A.A = \begin{bmatrix} \omega & \kappa \\ \eta & -\omega \end{bmatrix}.\begin{bmatrix} \omega & \kappa \\ \eta & -\omega \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \omega^2 + \kappa\eta & \omega\kappa - \omega\kappa \\ \omega\eta - \omega\eta & \eta\kappa + \omega^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \omega^2 + \kappa\eta & 0 \\ 0 & \omega^2 + \kappa\eta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

By equality of matrices, we have : $\omega^2 + \kappa\eta = 1$ and, $\omega^2 + \kappa\eta = 1$

$\therefore 1 - \omega^2 - k\eta = 0$.

Q10. **(a)** $A^2 = A$ ...(i)

Let $P = (I+A)^3 - 7A = (I+A)(I+A)(I+A) - 7A$

$\Rightarrow \quad = (I.I + I.A + A.I + A.A)(I+A) - 7A = (I + 2A + A^2)(I+A) - 7A$

$\Rightarrow \quad = (I+3A)(I+A) - 7A$ (By (i))

$\Rightarrow \quad = (I.I + I.A + 3A.I + 3A^2) - 7A$

$\Rightarrow \quad = I + A + 3A + 3A - 7A$

$\therefore \quad P = I$.

Can we use $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ formula here? Think. Also see next part (b).

(b) Here $A^2 = I = A.A$ ...(i)

Let $P = (A-I)^3 + (A+I)^3 - 7A$

$\Rightarrow P = A^3 - 3A^2I + 3A.I^2 - I^3 + A^3 + 3A^2I + 3A.I^2 + I^3 - 7A$

$\Rightarrow P = A^2A - 3A^2 + 3A - I + A^2A + 3A^2 + 3A + I - 7A$

By (i), $P = IA - 3I + 3A - I + IA + 3I + 3A + I - 7A$

$\Rightarrow P = A + A + 6A - 7A$

$\Rightarrow P = A$.

Note that, here we have used $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ and $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ formulae since, $AI = IA$ i.e., commutative nature holds well here.

Alternatively, let $P = (A-I)^3 + (A+I)^3 - 7A$

$\Rightarrow \quad = (A-I)(A-I)(A-I) + (A+I)(A+I)(A+I) - 7A$

$\Rightarrow \quad = (A.A - A.I - I.A + I.I)(A-I) + (A.A + A.I + I.A + I.I)(I+A) - 7A$

$\Rightarrow \quad = (A^2 - A - A + I)(A-I) + (A^2 + A + A + I)(I+A) - 7A$

$\Rightarrow \quad = (I - 2A + I)(A-I) + (I + 2A + I)(I+A) - 7A$ (By (i))

$\Rightarrow \quad = 2(-A+I)(A-I) + 2(A+I)(I+A) - 7A$

$\Rightarrow \quad = 2(-A.A + A.I + I.A - I.I) + 2(A.I + A.A + I.I + I.A) - 7A$

$\Rightarrow \quad = 2(-I + A + A - I) + 2(A + I + I + A) - 7A = 4(A-I) + 4(A+I) - 7A$

$\Rightarrow \quad = A$

$\therefore \quad P = A$.

Q11. $\begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix} = I_3$

$$\Rightarrow \begin{bmatrix} -2x+7x & 28x-28x & 14x-14x \\ 0 & 1 & 0 \\ -x+x & 14x-2-4x & 7x-2x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5x & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10x-2 & 5x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

By equality of matrices, we get :

$5x = 1 \Rightarrow x = \frac{1}{5}, 10x - 2 = 0$

$\therefore x = \frac{2}{10} = \frac{1}{5}$.

EXERCISE 1.4

Q01. We have $AC = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0-12+21 \\ -12+0+24 \\ 14+16+0 \end{bmatrix} = \begin{bmatrix} 9 \\ 12 \\ 30 \end{bmatrix}$,

and, $BC = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0-2+3 \\ 2+0+6 \\ 2-4+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ -2 \end{bmatrix}$.

Also $A + B = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 7 & 8 \\ -5 & 0 & 10 \\ 8 & -6 & 0 \end{bmatrix}$

$$\Rightarrow (A+B)C=\begin{bmatrix}0 & 7 & 8\\ -5 & 0 & 10\\ 8 & -6 & 0\end{bmatrix}\begin{bmatrix}2\\ -2\\ 3\end{bmatrix}=\begin{bmatrix}0-14+24\\ -10+0+30\\ 16+12+0\end{bmatrix}=\begin{bmatrix}10\\ 20\\ 28\end{bmatrix}\ldots(i)$$

$$\text{and, } AC+BC=\begin{bmatrix}9\\ 12\\ 30\end{bmatrix}+\begin{bmatrix}1\\ 8\\ -2\end{bmatrix}=\begin{bmatrix}10\\ 20\\ 28\end{bmatrix}\ldots(ii)$$

By (i) and (ii), it is clear that $(A+B)C=AC+BC$.

Q02. Let $P=\begin{bmatrix}2 & -1\\ 1 & 0\\ -3 & 4\end{bmatrix}$ and, $Q=\begin{bmatrix}-1 & -8 & -10\\ 1 & -2 & -5\\ 9 & 22 & 15\end{bmatrix}$.

Since order of P is '3 by 2' and that of matrix Q is '3 by 3' so, matrix A must be of order 2×3.

Let $A=\begin{bmatrix}u & v & w\\ x & y & z\end{bmatrix}$.

Now $PA=Q$

$$\therefore \begin{bmatrix}2 & -1\\ 1 & 0\\ -3 & 4\end{bmatrix}\begin{bmatrix}u & v & w\\ x & y & z\end{bmatrix}=\begin{bmatrix}-1 & -8 & -10\\ 1 & -2 & -5\\ 9 & 22 & 15\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}2u-x & 2v-y & 2w-z\\ u & v & w\\ -3u+4x & -3v+4y & -3w+4z\end{bmatrix}=\begin{bmatrix}-1 & -8 & -10\\ 1 & -2 & -5\\ 9 & 22 & 15\end{bmatrix}$$

By equality of matrices, we get : $2u-x=-1, 2v-y=-8, 2w-z=-10, u=1, v=-2, w=-5$, $-3u+4x=9, -3v+4y=22, -3w+4z=15$

On solving these equations simultaneously, we get : $u=1, v=-2, w=-5, x=3, y=4, z=0$.

Therefore, $A=\begin{bmatrix}1 & -2 & -5\\ 3 & 4 & 0\end{bmatrix}$.

Q03. $A=\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$, $B\begin{bmatrix}x & 1\\ y & -1\end{bmatrix}$

As $A^2+B^2=(A+B)^2$

$\Rightarrow A^2+B^2=(A+B)(A+B)$

$\Rightarrow A^2+B^2=A^2+AB+BA+B^2$.

SOLUTIONS

Chapter 01
(MCQ - MATHMISSION)

Q01. Since matrices of same order can be added only. So, A and B must be of some order.
Also AB is defined as well so, the number of columns in A must be same as the number of rows in B. Clearly from the given options, it can be concluded that (d) is correct.

Q02. Here $AA^T = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ 5 & 10 \end{bmatrix} = 5\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$.

Q03. As $|adj.A| = |A|^{n-1}$ where n is order of A so, $|adj.A| = |A|^{3-1} = (4)^2 = 16$.

Q04. $A^2 = AA = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

So, $A^4 = A^2A^2 = I.I$

$\Rightarrow A^4 = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Q05. Note that $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = -I_3$ so, $|A| = |-I_3| = (-1)^3 |I_3| = (-1)^3(1) = -1$

Also, $|A||adjA| = |A||A|^{3-1} = |A|^3 = (-1)^3 = -1$.

Q06. $A^2 - B^2 = (A-B)(A+B)$

$\Rightarrow A^2 - B^2 = AA + AB - BA - BB$

$\Rightarrow A^2 - B^2 = A^2 + AB - BA - B^2$

$\Rightarrow O = AB - BA$

$\therefore AB = BA$.

Q07. Recall that, $|adj.A| = |A|^{n-1}$, n is order of A

If A is singular matrix then, $|A| = 0$ so, $|adj.A| = 0$

Therefore, adj.A is singular matrix as well.

Q08. As the order of A, B and C are 4×3, 5×4 and 3×7 respectively so, the order of A', B' and C' will be 3×4, 4×5 and 7×3 respectively.
Now order of $A'\times B'$ is 3×5
So, the order of $C'(A'\times B')$ is 7×5.

Q09. $|A| = \begin{vmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{vmatrix} = a\times a\times a = a^3$

Now $|adj.A| = |A|^{3-1} = (a^3)^2 = a^6$.

Q10. $A^2 - A + I = O$

$\Rightarrow A^{-1}A^2 - A^{-1}A + A^{-1}I = A^{-1}O$

$\Rightarrow A^{-1}AA - I + A^{-1} = O$

$\Rightarrow A^{-1} = I - IA$

$\Rightarrow A^{-1} = I - A$.

Q11. $|A^3| = 125$

$\Rightarrow |A|^3 = 125$

$\Rightarrow |A| = \sqrt[3]{125}$

$\Rightarrow \begin{vmatrix} \alpha & 2 \\ 2 & \alpha \end{vmatrix} = 5$

$\Rightarrow \alpha^2 - 4 = 5$

$\Rightarrow \alpha^2 = 9$

$\therefore \alpha = \pm 3$.

Q12. Order of the product of the matrices is 1×1.

Q13. Recall that, $A.(adj.A) = |A|\, I_n$ where n is order of A

That is, $A.(adj.A) = |A|\, I_n$ where n is order of A

As $A.(adj.A) = k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = k\,I_2$ so, $|A| = k$.

Now $\begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = k$

$\Rightarrow k = \cos^2 x - (-\sin^2 x)$

$\therefore k = \cos^2 x + \sin^2 x = 1$.

Q14. Since $AA^T = I$

Now $|AA^T| = |I|$

$\Rightarrow |A||A^T| = 1$

$\Rightarrow |A||A| = 1$

$\Rightarrow |A|^2 = 1$

$\Rightarrow |A| = -1, 1$.

Q15. $(AB^{-1}C)^{-1} = C^{-1}(B^{-1})^{-1}A^{-1} = C^{-1}BA^{-1}$.

Q16. Since all the diagonal elements can be multiplied to get the det. value of a scalar matrix.

So, $|A| = k \times k \times k \times ...\ n$ times (as the order of A is $n \times n$

$\therefore\ |A| = k^n$.

Q17. $|3AB| = 3^3|AB| = 27|A||B| = 27(-1)(3) = -81$.

SOLUTIONS

ASSERTION-REASON

Type Questions

Unit 1 (Relations & Functions)

Relations & Functions, Inverse Trig. Functions

Q01. (a) Note that $\frac{1}{2} > \left(\frac{1}{2}\right)^2 \Rightarrow \left(\frac{1}{2}, \frac{1}{2}\right) \notin R$.

Hence, R is not reflexive.

Q02. (d) Let $a, b, c \in \mathbb{R}$. Let $(a, b) \in R$ and $(b, c) \in R$.

Put $a = 1,\ b = -\frac{1}{2},\ c = -1$.

Note that $\left(1, -\frac{1}{2}\right) \in R$ as $1 + 1\left(-\frac{1}{2}\right) = \frac{1}{2} > 0$.

Similarly, $\left(-\frac{1}{2}, -1\right) \in R$ as $1 + \left(-\frac{1}{2}\right)(-1) = \frac{3}{2} > 0$

But $(1, -1) \notin R$ as $1 + (1)(-1) = 0 > 0$ is false.

Hence, R is not transitive.

Q03. (d) Since $|a - b| \le \frac{1}{2}$ implies, $|-(b - a)| \le \frac{1}{2}$ i.e., $|b - a| \le \frac{1}{2}$.

That is, aRb implies bRa $\forall a, b \in Q$.

Therefore, R is symmetric relation.

Q04. (a) $R = \{(T_1, T_2) : T_1 \sim T_2\}$, $R : T \to T$.

Note that R is reflexive, since every triangle is similar to itself, that is $(T_1, T_1) \in R\ \forall T_1 \in T$.

Further, $(T_1, T_2) \in R \Rightarrow T_1$ is similar to $T_2 \Rightarrow T_2$ is similar to $T_1 \Rightarrow (T_2, T_1) \in R$.

Hence, R is symmetric.

Moreover, $(T_1, T_2), (T_2, T_3) \in R \Rightarrow T_1$ is similar to T_2 and T_2 is similar to T_3.

$\Rightarrow T_1$ is similar to $T_3 \Rightarrow (T_1, T_3) \in R$.

So R is transitive.

Therefore, R is an equivalence relation.

Q05. (a) A relation R on A is identity relation iff $R = \{(a, b) : a \in A, b \in A \text{ and } a = b\}$ that is, the identity relation R contains only elements of the type $(a, a)\ \forall a \in A$ and it must **not** contain any other element.

While in case of reflexive relation, R must contain $(a, a)\ \forall a \in A$ but it may contain other elements as well.

Unit I - Relations & Functions

Q01. **(i)** No. of Reflexive relations defined on a set of n elements $= 2^{n(n-1)}$.

Therefore, no. of reflexive relations defined on set A having 5 elements $= 2^{5\times 4} = 2^{20}$.

(ii) As $(x, x) \in R$ for all $x \in A$, when x is either boy or girl.

So, R is reflexive.

Let $(x, y) \in R$ that is, x and y are of same sex.

That means, y and x are also of same sex.

This implies, $(y, x) \in R$. So, R is symmetric.

Also let $(x, y) \in R$ and $(y, z) \in R$.

That means, x and y are of same sex; y and z are of same sex.

Clearly, x and z will also be of same sex.

That implies, $(x, z) \in R$. So, R is transitive.

Therefore, R is equivalence relation.

(iii) No. of Symmetric relations defined on a set of n elements $= 2^{\frac{n(n+1)}{2}}$.

Therefore, no. of symmetric relations defined on set A having 5 elements $= 2^{\frac{5\times 6}{2}} = 2^{15}$

Therefore, no. of symmetric relations defined on set B having 4 elements $= 2^{\frac{4\times 5}{2}} = 2^{10}$

Hence, the required difference is $= 2^{15} - 2^{10} = 2^{10}(31)$.

(iv) For the element $a_1 \in A$, we have different images under R as, $(a_1, b_1), (a_1, b_2) \in R$.

So, R is not a function.

(v) If A and B are two sets having m and n elements respectively such that $m \geq n$, then total no. of onto functions from set A to set B is $= \sum_{r=0}^{n} (-1)^r \times {}^nC_r \times (n-r)^m$.

Here $n(A) = 5,\ n(B) = 4$.

So, the number of onto functions from set A to set B $= \sum_{r=0}^{4} (-1)^r \times {}^4C_r \times (4-r)^5$

$\Rightarrow \quad = (-1)^0 \times {}^4C_0 \times (4-0)^5 + (-1)^1 \times {}^4C_1 \times (4-1)^5 + (-1)^2 \times {}^4C_2 \times (4-2)^5$
$\quad + (-1)^3 \times {}^4C_3 \times (4-3)^5 + (-1)^4 \times {}^4C_4 \times (4-4)^5$

$\Rightarrow \quad = 1 \times 1 \times (4)^5 + (-1) \times 4 \times (3)^5 + 1 \times 6 \times (2)^5 + (-1) \times 4 \times 1 + 1 \times 1 \times 0$

$\Rightarrow \quad = 1024 - 972 + 192 - 4 = 240$.

Q02. **(i)** Here, perfectness 'y' has an inverse square relationship with dishonesty 'x'.

Also, $y = 1,\ x = 1$.

So, $y = \dfrac{1}{x^2},\ x \neq 0$.

(ii) As $y = \dfrac{1}{x^2},\ x \neq 0$

So $x \in (0,\infty)$ implies, y must be in $(0,\infty)$.

(iii) Let $\alpha, \beta \in (0,\infty)$. Let $f(\alpha) = f(\beta)$.

Then, $\frac{1}{\alpha^2} = \frac{1}{\beta^2}$

$\Rightarrow \alpha^2 - \beta^2 = 0 \quad \Rightarrow (\alpha-\beta)(\alpha+\beta) = 0$

As $(\alpha+\beta) \neq 0$ as $\alpha, \beta \in (0,\infty)$ so, $(\alpha-\beta) = 0$

That is, $\alpha = \beta$.

So, $y = f(x) = \frac{1}{x^2}$, $x \neq 0$ is a one-one function.

(iv) Refer to (i).

We have $y = \frac{1}{x^2}$

When $x = 4$, $y = \frac{1}{4^2} = \frac{1}{16}$ and, when $x = 2$, $y = \frac{1}{2^2} = \frac{1}{4}$.

Therefore, the change in level of perfection is $\Delta y = \frac{1}{4} - \frac{1}{16} = \frac{3}{16}$.

Q03. **(i)** Clearly the domain of sine function is the set of all real numbers i.e., R.
And, its range is [–1, 1].

(ii) When suitable restriction is imposed on the domain of sine function, it becomes invertible. Therefore, the sine function becomes one-one and onto both.

(iii) The range of principal value branch for $\sin^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

(iv) Referring to the graphs, the range of sine inverse function other than its principal value branch will be $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$.

Note that, if we were not asked to answer with the restriction of the graph shown above, then the many answers could have been possible e.g., $\left[-\frac{3\pi}{2}, -\frac{\pi}{2}\right]$, $\left[\frac{3\pi}{2}, \frac{5\pi}{2}\right]$, $\left[\frac{5\pi}{2}, \frac{7\pi}{2}\right]$ etc.

(v) $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \sin^{-1}\sin\left(-\frac{\pi}{4}\right) = -\frac{\pi}{4}$.

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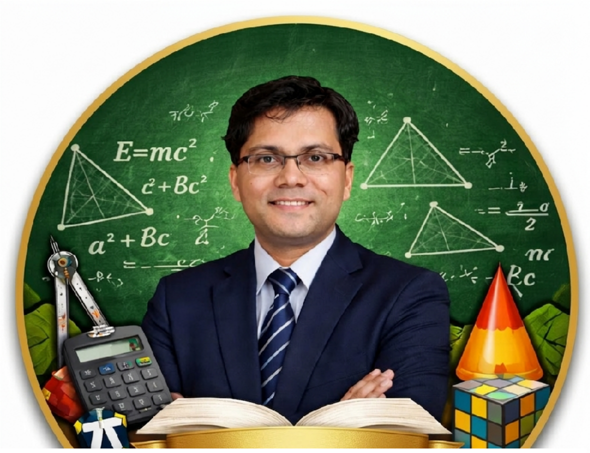

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ABOUT THE AUTHOR

O.P. GUPTA having taught math passionately over a decade, has devoted himself to this subject. Every book, study material or practice sheets, tests he has written, tries to teach serious math in a way that allows the students to learn math without being afraid. Undoubtedly his mathematics books are best sellers on amazon and Flipkart.

His resources have helped students and teachers for a long time across the country. He has contributed in CBSE Question Bank (issued in April 2021). Mr Gupta has been invited by many educational institutions for hosting sessions for the students of senior classes. Being qualified as an electronics & communications engineer, he has pursued his graduation later on with mathematics from University of Delhi due to his passion towards mathematics. He has been honored with the prestigious INDIRA AWARD by the Govt. of Delhi for excellence in education.

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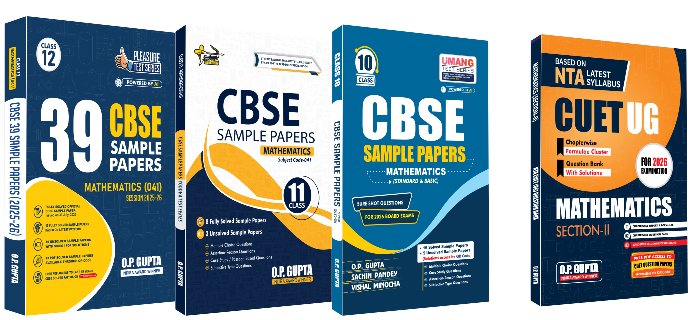

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